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The Operational Equations of State.
5: The APΛ−Equation of State

by Michael Grinfeld

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5: The $\mathbf{AP\Lambda-}$Equation of State

Michael Grinfeld
Weapons and Materials Research Directorate, ARL

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14. ABSTRACT

The widely spread hydrocode uses the specific volume V and the specific internal energy E as independent thermodynamic variables. Because of that fact, the thermodynamically consistent implementation of the hydrocode requires the complete equation of state (EOS) in the form $S = S(V, E)$, where S is the specific entropy. We suggest a complete EOS for hydrocode which is an operator of three functions of one variable each: $A(V)$, $P(E)$, and $\Lambda(V)$.

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Contents

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1. Introduction and Definition

This report is a further development of previous reports (*1–4*) based on the concept of the operational equations of state. It is suggested that the $AP\Lambda-$ equation of state (EOS) depends on three functions of one variable each that can be presented in the following simple form:

$$S(V,E) = A(V) + \int_{E_*}^{E} \frac{d\xi}{P(\xi) + \Lambda(V)} \, .$$

(1)

Thus, in accordance with the general paradigm of the operational EOS, the $AP\Lambda-EOS$ is the integral operator transforming the functions $A(V)$, $P(E)$, and $\Lambda(V)$ into the third function of two variables, called entropy $S(V,E)$.

In their own turn, the functions $A(V)$, $P(E)$, and $\Lambda(V)$ are also certain operators of experimental data. Eventually, the $AP\Lambda-EOS$ can be treated as the operators transforming experimental measurements into EOS rather than algebraic functions.

The postulated EOS equation 1 is fixed in terms of the functions $A(V)$, $P(E)$, and $\Lambda(V)$. We call it the skeleton of the operational EOS. However, our goal is to present the operational EOS as an operator whose arguments are not abstract functions but experimentally measured data. Those ultimate operational EOS depend upon the measuring tools available. By using the same skeleton but different measuring tools and procedures the ultimate operational EOS will be different. In particular, the ultimate $AP\Lambda-EOS$ depends on the specific types of the available measurements–isochoric, adiabatic, isothermal, etc. The general intermediate structure equation 1 guarantees that with any options of the chosen experiments and measuring devices, the ultimate $AP\Lambda-EOS$ will be complete and thermodynamically consistent.

We remind the reader that completeness means that the EOS contains the full thermodynamic information about the substance and any required thermodynamic potential or function can be extracted by standard algebraic and calculus operations and without any additional physical measurements. Thermodynamic consistency means that the $AP\Lambda-EOS$ is totally consistent with the first and second laws of thermodynamics. Remember that the most popular and widespread EOS in the high-pressure and shock-wave analysis community, the Mie-Gruneisen EOS, is neither complete nor necessarily thermodynamically consistent with other assumptions. Generally speaking, the $AP\Lambda-EOS$ is incompatible with the Mie-Gruneizen EOS (*1, 2*).

Roughly speaking, there are no constraints in the choice of the function $S = S(V, E)$ from the standpoint of thermodynamic consistency. In other words, there are no modeling assumptions when working with the most general function $S = S(V, E)$. At the same time, the skeleton equation 1 imposes certain constraints on the substances under consideration. The assumptions can be dictated by mathematical or computational conveniences or generalization of a simple classical model. The assumptions can also be triggered by theoretical or experimental physical analysis. In all these cases of modeling assumptions there is the risk of thermodynamic inconsistency, insufficiency, or redundancy of generated models. The risk is greater when dealing with the sophisticated mathematical forms like equation 1.

The assumption equation 1 has a clear physical formulation as it implies that the specific heat capacity C at constant volume V is a function of the internal energy density E only: $C = C(E)$. The opposite statement is also valid: assuming that the heat capacity is a function of the internal energy only, one can deduce that the specific entropy S can be presented in the general form equation 1. From that standpoint, the $AP\Lambda - EOS$ reminds the classical Boyle-Mariotte-Gay-Lussac model (when the heat capacity must be function of the absolute temperature T only) and the Mie-Gruneisen EOS (when the heat capacity must be a function of the entropy S only). At the same time, the $AP\Lambda - EOS$ obviously differs from both previously mentioned classical models with one important exception. This exception concerns the model with the constant heat capacity coined in (4) as the Dulong-Petit model. In this case, all four models are equivalent to each other.

2. Thermodynamic Identities for the $AP\Lambda$ EOS

By applying standard thermodynamic identities to the definition equation 1 of the $AP\Lambda - EOS$, we get the following formulas for the absolute temperature T, pressure P, and heat capacity C_V:

$$T(V, E) = P(E) + \Lambda(V) . \tag{2}$$

$$P(V, E) = \left[P(E) + A(V) \right] \left\{ A'(V) - \Lambda'(V) \int_{E_*}^{E} d\xi \frac{1}{\left[P(\xi) + \Lambda(V) \right]^2} \right\} . \tag{3}$$

$$C_V(V, E) = \frac{1}{P'(E)} = C_V(E) . \tag{4}$$

According to equation 2, the absolute temperature for the $AP\Lambda - EOS$ appears to be the sum of the function $P(E)$ of the internal energy density E and of the function $\Lambda(V)$ of the specific volume V. Equation 4 also states that the specific heat capacity function C_V appears to be the function of the specific internal energy E only.

The Gruneisen function $\Gamma(V,E)$ for the $AP\Lambda - EOS$ is given by the following formula:

$$\frac{\Gamma(V,E)}{V} = P'(E)A'(V) - \frac{1}{P'(E^*)}\frac{\Lambda'(V)P'(E)}{P(E^*)+\Lambda(V)} + \int_{E_*}^{E} d\xi \frac{\Lambda'(V)P'(E)}{P(\xi)+\Lambda'(V)}\frac{P''(\xi)}{[P'(\xi)]^2} \quad . \tag{5}$$

Combining equations 4 and 5, we get

$$\frac{\Gamma(V,E)}{V} = \frac{A'(V)}{C_V(E)} - \frac{C_V(E^*)}{C_V(E)}\frac{\Lambda'(V)}{P(E^*)+\Lambda(V)} - \frac{1}{C_V(E)}\int_{E_*}^{E} d\xi \frac{C_V'(\xi)\Lambda'(V)}{P(\xi)+\Lambda(V)} \quad . \tag{6}$$

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3.   Recovery of the A, P, Λ Functions From Physical Experiments

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3.1   Recovery of the Function $P(E)$ From Isochoric Heating

The way of recovery of the function $P(E)$ is quite straightforward. To that end, the isochoric heating is the most relevant source of information. In the following, all the parameters relating to the ambient state will be supplied with the asterisk mark.

Let $T_V^* = T_V^*(E)$ be the temperature dependence upon the internal energy density at the ambient volume $V = V^*$. By definition, the internal energy change in the isochoric process is equal to the heat supply Q. Thus, the determination of the function $T_V^* = T_V^*(E)$ requires a thermometer and a device measuring the heat supply. Then, equation 2 allows us to recover the function $P(E)$:

$$P(E) = T_V^*(E) - \Lambda(V^*) = T_V^*(E) - \Lambda^* . \tag{7}$$

We can now rewrite the general $AP\Lambda - EOS$ 1 as follows:

$$S(V,E) = A(V) + \int_{E_*}^{E} \frac{d\xi}{T_V^*(\xi) + \Lambda(V) - \Lambda^*} , \tag{8}$$

and equations 2 and 3 as follows:

$$T(V,E) = T_V^*(E) + \Lambda(V) - \Lambda^* \tag{9}$$

and

$$P(V,E) = \left[T_V^*(E) + \Lambda(V) - \Lambda^*\right]\left\{A'(V) - \Lambda'\int_{E_*}^{E} \frac{d\xi}{\left[T_V^*(\xi) + \Lambda(V) - \Lambda^*\right]^2}\right\}. \tag{10}$$

3.2 Recovery of the Functions $A(V)$ and $\Lambda(V)$ From the Quasi-Static Ambient Isotherm

3.2.1 Formula of the Potential

Let $E_T^* = E_T^*(V)$ be the equation of the ambient isotherm $T = T^*$. Then, equation 9 gives the following formula of $\Lambda(V)$:

$$\Lambda(V) - \Lambda^* = T^* - T_V^*\left(E_T^*(V)\right). \tag{11}$$

Substituting equation 11 in equation 8, we get

$$S(V,E) = A(V) + \int_{E_*}^{E} \frac{d\xi}{T_V^*(\xi) + T^* - T_V^*\left(E_T^*(V)\right)}. \tag{12}$$

Let $S_T^* = S_T^*(V)$ be the values of the entropy on the ambient isotherm $E_T^* = E_T^*(V)$. Then we can rewrite equation 12 as follows:

$$A(V) = S_T^*(V) - \int_{E_*}^{E_T^*(V)} \frac{d\xi}{T_V^*(\xi) + T^* - T_V^*\left(E_T^*(V)\right)}. \tag{13}$$

Inserting equation 13 in equation 12, we get

$$S(V,E) = S_T^*(V) + \int_{E_T^*(V)}^{E} \frac{d\xi}{T_V^*(\xi) + T^* - T_V^*\left(E_T^*(V)\right)}. \tag{14}$$

This formula should be augmented with

$$E_T^*(V) - E^* = -\int_{V_*}^{V} d\varpi P_T^*(\varpi) + T^*\left[S_T^*(V) - S^*\right],$$

implied by

$$\frac{dE_T^*(V)}{dV} = -P_T^*(V) + T^* \frac{dS_T^*(V)}{dV}.$$

3.2.2 Temperature and Pressure Formulas

Using equation 14, we get the following relationships for the absolute temperature and pressure:

$$T(V,E) = T_V^*(E) + T^* - T_V^*\left(E_T^*(V)\right). \tag{15}$$

$$\frac{P(V,E)}{T_V^*(E)+T^*-T_V^*\left(E_T^*(V)\right)}=\frac{dS_T^*(V)}{dV}-\frac{1}{T^*}\frac{dE_T^*(V)}{dV}+$$

$$\int_{E_T^*(V)}^{E} d\xi \frac{dT_V^*\left(E_T^*(V)\right)/dV}{\left[T_V^*(\xi)+T^*-T_V^*\left(E_T^*(V)\right)\right]^2} \quad .$$

(16)

3.3 Recovery of the Functions $A(V)$ and $\Lambda(V)$ From the Quasi-Static Ambient Adiabata

3.3.1 Formula of the Potential

Let $E_S^* = E_S^*(V)$ be the equation of the ambient adiabata $S = S^*$ in the (V,E) space. Then, we get

$$S(V,E)=S^* + \int_{E_S^*(V)}^{E} \frac{d\xi}{T_V^*(\xi)+\Lambda(V)-\Lambda^*} \quad . \tag{17}$$

Let $T_S^* = T_S^*(V)$ be the values of temperature, corresponding to the ambient adiabata $S = S^*$. Then, using equation 9, we get

$$\lambda(V)-\lambda^* = T_S^*(V)-T_V^*\left(E_S^*(V)\right) \tag{18}$$

Inserting equation 18 into equation 17, we get

$$S(V,E)=S^* + \int_{E_S^*(V)}^{E} \frac{d\xi}{T_V^*(\xi)+T_S^*(V)-T_V^*\left(E_S^*(V)\right)} \quad . \tag{19}$$

3.3.2 Temperature and Pressure

Using equation 19, we get

$$T(V,E)=T_V^*(E)+T_S^*(V)-T_V^*\left(E_S^*(V)\right), \tag{20}$$

and

$$P(V,E)=-\frac{T_S^*(V)+T_V^*(E)-T_V^*\left(E_S^*(V)\right)}{T_S^*(V)}\frac{dE_S^*(V)}{dV}-$$

$$\int_{E_S^*(V)}^{E} d\xi \frac{T_V^*(E)+T_S^*(V)-T_V^*\left(E_S^*(V)\right)}{\left[T_V^*(\xi)+T_S^*(V)-T_V^*\left(E_S^*(V)\right)\right]^2}\frac{d\left[T_S^*(V)-T_V^*\left(E_S^*(V)\right)\right]}{dV} \quad .$$

(21)

3.4 Recovery of the Functions $A(V)$ and $\Lambda(V)$ From the Quasi-Static Ambient Isotherm and the Hugoniot Adiabata

3.4.1 Formula of the Potential

When dealing with this sort of available measurements we can recover the function $\lambda(V)$ as noted before and deal further with the equation containing the unknown function $\alpha(V)$ only:

$$S(V,E) = A(V) + \int_{E_*}^{E} d\varepsilon K(V,\varepsilon), \tag{22}$$

where

$$K(V,E) \equiv \frac{1}{T_V^*(E) + T^* - T_V^*\left(E_T^*(V)\right)}. \tag{23}$$

It is important to realize that $K(V,\xi)$ should be treated as the already recovered function.

Using equation 22, we get the following formulas of the absolute temperature and pressure:

$$T(V,E) = \frac{1}{K(V,E)}, \quad P(V,E) = -\frac{1}{K(V,E)}A'(V) - \frac{1}{K(V,E)}\int_{E_*}^{E} d\varepsilon \frac{\partial K(V,\varepsilon)}{\partial V}. \tag{24}$$

Let $E_H(V)$ be the energy measurements on the Hugoniot adiabata. Then, using the equation of the Hugoniot adiabata we arrive at the following formula for the function $A(V)$:

$$A(V) = A^* + \int_{V^*}^{V} d\eta K\left(\eta, E_H(\eta)\right)\frac{2E_H(\eta) - 2E_* + P_*(\eta - V_*)}{\eta - V_*} - \int_{V^*}^{V} d\eta \int_{E_*}^{E_H(\eta)} d\varepsilon \frac{\partial K(\eta,\varepsilon)}{\partial \eta}, \tag{25}$$

which allows us the present the operational EOS for hydrocode in the following form:

$$S(V,E) = S^* + \int_{E_H(V)}^{E} d\varepsilon K(V,\varepsilon) + \\ \int_{V^*}^{V} d\eta K\left(\eta, E_H(\eta)\right)\left[\frac{2E_H(\eta) - 2E_* + P_*(\eta - V_*)}{\eta - V_*} + \frac{dE_H(\eta)}{d\eta}\right]. \tag{26}$$

3.4.2 Temperature and Pressure

The formula in equation 26 implies two formulas:

$$T(V,E) = \frac{1}{K(V,E)} = T_V^*(E) + T^* - T_V^*\left(E_T^*(V)\right), \tag{27}$$

and

$$P(V,E) = \int_{E_H(V)}^{E} d\varepsilon \frac{1}{K(V,E)} \frac{\partial K(V,\varepsilon)}{\partial V} + \frac{K(V,E_H(V))}{K(V,E)} \frac{2E_H(V) - 2E_* + P_*(V - V_*)}{V - V_*}. \qquad (28)$$

3.5 Recovery of the Functions $A(V)$ and $\Lambda(V)$ From the Hugoniot Energy and Temperature Measurements

3.5.1 Formula of the Potential

Let $E_H(V)$ and $T_H(V)$ be the energy and temperature measurements on the Hugoniot adiabata. Then, we arrive at the following EOS:

$$S(V,E) = A(V) + \int_{E_*}^{E} d\varepsilon K_H(V,\varepsilon), \ K_H(V,E) \equiv \frac{1}{T_V^*(E) + T_H(V) - T_V^*(E_H(V))}. \qquad (29)$$

We arrive at the situation considered previously. Therefore, the formula in equation 26 should be modified as follows:

$$S(V,E) = S^* + \int_{E_H(V)}^{E} d\varepsilon K_H(V,\varepsilon) + \\ \int_{V^*}^{V} d\eta K_H(\eta, E_H(\eta)) \left[\frac{2E_H(\eta) - 2E_* + P_*(\eta - V_*)}{\eta - V_*} + \frac{dE_H(\eta)}{d\eta} \right], \qquad (30)$$

or else

$$S(V,E) = S^* + \int_{E_H(V)}^{E} d\varepsilon \frac{1}{T_V^*(\varepsilon) + T_H(V) - T_V^*(E_H(V))} + \\ \int_{V^*}^{V} d\eta \frac{1}{T_H(\eta)} \left[\frac{2E_H(\eta) - 2E_* + P_*(\eta - V_*)}{\eta - V_*} + \frac{dE_H(\eta)}{d\eta} \right]. \qquad (31)$$

3.5.2 Temperature and Pressure

For temperature and pressure, we have the following recovery formulas:

$$T(V,E) = \frac{1}{K_H(V,E)} = T_V^*(E) + T_H(V) - T_V^*(E_H(V)), \qquad (32)$$

and

$$P(V,E) = \int_{E_H(V)}^{E} d\varepsilon \frac{1}{K_H(V,E)} \frac{\partial K_H(V,\varepsilon)}{\partial V} + \frac{K_H(V,E_H(V))}{K_H(V,E)} \frac{2E_H(V) - 2E_* + P_*(V - V_*)}{V - V_*}, \quad (33)$$

respectively.

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4. The Constant Heat Capacity Case

This case was considered earlier in (*4*), using a different approach. Here this important case is analyzed from the more general point of view.

4.1 One Important Relationship

We begin with the relationship

$$\int_{E_*}^{E} \frac{d\xi}{\rho(\xi) + \lambda(V)} =$$
$$\frac{\ln[\rho(E) + \lambda(V)]}{\rho'(E)} - \frac{\ln[\rho(E_*) + \lambda(V)]}{\rho(E_*)} + \int_{E_*}^{E} d\xi \frac{\rho''(\xi)}{\rho'^2(\xi)} \ln[\rho(\xi) + \lambda(V)], \quad (34)$$

implied by the following chain:

$$\int_{E_*}^{E} \frac{d\xi}{\rho(\xi) + \lambda(V)} = \int_{E_*}^{E} \frac{1}{\rho'(\xi)} \frac{d_\xi[\rho(\xi) + \lambda(V)]}{\rho(\xi) + \lambda(V)} = \int_{E_*}^{E} \frac{1}{\rho'(\xi)} d_\xi \ln[\rho(\xi) + \lambda(V)] =$$
$$\ln \frac{\rho(\xi) + \lambda(V)}{\rho'(E)} - \ln \frac{\rho(E_*) + \lambda(V)}{\rho'(E_*)} - \int_{E_*}^{E} \ln[\rho(\xi) + \lambda(V)] d_\xi \frac{1}{\mathrm{P}'(\xi)} =$$
$$\frac{\ln[\rho(\xi) + \lambda(V)]}{\rho'(E)} - \frac{\ln[\rho(E_*) + \lambda(V)]}{\rho'(E_*)} + \int_{E_*}^{E} d\xi \frac{\rho''(\xi)}{\rho'^2(\xi)} \ln[\rho(\xi) + \lambda(V)] \qquad .$$

We can rewrite equation 34 as follows:

$$\int_{E_*}^{E} \frac{d\xi}{\rho(\xi) + \lambda(V)} = \ln \frac{[\rho(\xi) + \lambda(V)]^{\frac{1}{\rho'(E)}}}{[\rho(E_*) + \lambda(V)]^{\frac{1}{\rho'(E_*)}}} + \int_{E_*}^{E} d\xi \frac{\rho''(\xi)}{\rho'^2(\xi)} \ln[\rho(\xi) + \lambda(V)]. \quad (35)$$

When $\rho'(E) \equiv ME + N$, where M and N are constants, using equation 34 we get

$$\int_{E_*}^{E} \frac{d\xi}{\rho(\xi)+\lambda(V)} = \frac{1}{M}\ln\frac{E+\tilde{\Lambda}(V)}{E_*+\tilde{\Lambda}(V)}, \tag{36}$$

as it follows from the chain

$$\int_{E_*}^{E} \frac{d\xi}{\rho(\xi)+\lambda(V)} = \frac{1}{M}\ln\frac{ME+N+\lambda(V)}{ME_*+N+\lambda(V)} = \frac{1}{M}\ln\frac{E+\tilde{\lambda}(V)}{E_*+\tilde{\lambda}(V)}.$$

By inserting equation 36 in equation 1, we get the particularly simple formula of the complete EOS:

$$S(V,E) = \alpha(V) + \frac{1}{M}\ln\frac{E+\tilde{\lambda}(V)}{E_*+\tilde{\lambda}(V)}, \tag{37}$$

or else

$$S(V,E) = \tilde{\alpha}(V) + \frac{1}{M}\ln\left[E+\tilde{\lambda}(V)\right]. \tag{38}$$

With the help of the thermodynamic identities for $S(V,E)$, the relationship (equation 38) allows us to get the relationships for the absolute temperature and pressure

$$T(V,E) = M\left[E+\tilde{\lambda}(V)\right], \tag{39}$$

and

$$P(V,E) = M\tilde{\alpha}'(V)E + M\tilde{\alpha}'(V)\tilde{\lambda}(V) + \tilde{\lambda}'(V), \tag{40}$$

the last of which is implied by the chain:

$$P(V,E) = M\left[E+\tilde{\lambda}(V)\right]\left\{\tilde{\alpha}'(V) + \frac{\tilde{\lambda}'(V)}{M\left[E+\tilde{\lambda}(V)\right]}\right\} = \tag{41}$$

$$M\left[E+\tilde{\lambda}(V)\right]\tilde{\alpha}'(V) + \tilde{\lambda}'(V) = M\tilde{\alpha}'(V)E + M\tilde{\alpha}'(V)\tilde{\lambda}(V) + \tilde{\lambda}'(V).$$

In the case of constant heat capacity, equations 2 and 4 imply

$$\mathrm{H}(E) \equiv \mathrm{P}(E) = \int_{E_*}^{E} d\eta C^{-1} = \frac{1}{C}(E-E_*) = T_V^*(E) - \lambda^*, \tag{42}$$

whereas equation 1 reads

$$S(V,E) = \alpha(V) + C\int_{E_*}^{E} \frac{d\xi}{\xi - E_* + C\lambda(V)} = \alpha(V) + C\ln\frac{E-E_*+C\lambda(V)}{C\lambda(V)}. \tag{43}$$

The thermodynamic identities 1 and 3 can be reduced to the form

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$$T(V,E) = \frac{1}{C}(E - E_*) + \lambda(V) = \frac{E - E_* + C\lambda(V)}{C} = T_L(V,E), \tag{44}$$

$$P(V,E) = \alpha'(V)T_L(V,E) - \frac{\lambda'(V)}{\lambda(V)}(E - E_*), \tag{45}$$

the last of which can be also rewritten as follows:

$$P(V,E) = (E - E_*)\left[\frac{1}{C}\alpha'(V) - \frac{\lambda'(V)}{\lambda(V)}\right] - \alpha'(V)\lambda(V). \tag{46}$$

4.2   Recovery of the Functions α(V) and λ(V) From the Quasi-Static Ambient Isotherm

This case in general is described by the relationship 14. Using equation 42, we can rewrite equation 15 as follows:

$$S(V,E) = S_T^*(V) + C\ln\frac{E - E_T^*(V) + CT^*}{CT^*}. \tag{47}$$

Equation 43 implies the following recovery equations for the absolute temperature,

$$T(V,E) = \frac{E - E_T^*(V) + CT^*}{C}, \tag{48}$$

and pressure

$$P(V,E) = \frac{E - E_T^*(V) + CT^*}{C}\frac{dS_T^*(V)}{dV} - \frac{dE_T^*(V)}{dV}. \tag{49}$$

4.3   Recovery of the Functions α(V) and λ(V) From the Quasi-Static Ambient Adiabat

This data is recovered by the formula equation 19. Combining it with equation 42, we get the following complete EOS:

$$S(V,E) = S^* + C\ln\frac{E - E_S^*(V) + CT_S^*(V)}{CT_S^*(V)}. \tag{50}$$

The formula (equation 50) implies the following incomplete EOS for the absolute temperature

$$T(V,S) = \frac{E - E_S^*(V) + CT_S^*(V)}{C}, \tag{51}$$

and pressure

$$P(V,E) = -E_S^{*'}(V) - \left[E - E_S^*(V)\right]\frac{T_S^{*'}(V)}{T_S^*(V)}, \tag{52}$$

the last of which can be rewritten as

$$P(V,E) = -E\frac{d\ln T_S^*(V)}{dV} - T_S^*(V)\frac{d}{dV}\frac{E_S^*(V)}{T_S^*(V)}. \tag{53}$$

4.4 Recovery of the Functions $\alpha(V)$ and $\lambda(V)$ From the Quasi-Static Ambient Isotherm and the Hugoniot Adiabata

The potential $S(V,E)$ and the kernel $K(V,E)$ are given by the formulas:

$$S(V,E) = S^* + C\ln\frac{E - E_T^*(V) + CT^*}{CT^*} +$$

$$C\int_{V^*}^{V} d\eta\,\frac{\dfrac{2E_H(\eta) - 2E_* + P_*(\eta - V_*)}{\eta - V_*} + \dfrac{dE_T^*(\eta)}{d\eta}}{E_H(\eta) - E_T^*(\eta) + CT^*}, \tag{54}$$

and

$$K(V,E) \equiv \frac{C}{E - E_T^*(V) + CT^*}. \tag{55}$$

Respectively, the temperature and pressure are given by the formulas

$$T(V,E) = \frac{1}{K(V,E)} = \frac{E - E_T^*(V) + CT^*}{C} = T^* + \frac{E - E_T^*(V)}{C}, \tag{56}$$

and

$$P(V,E) = \frac{dE_T^*(V)}{dV}\ln\frac{E - E_T^*(V) + CT^*}{E_H(V) - E_T^*(V) + CT^*} +$$

$$\frac{E - E_T^*(V) + CT^*}{V - V_*}\frac{2E_H(V) - 2E_* + P_*(V - V_*)}{E_H(V) - E_T^*(V) + CT^*}. \tag{57}$$

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4.5 Recovery of the Functions $\alpha(V)$ and $\lambda(V)$ From the Hugoniot Energy and Temperature Measurements

By using equation 31, we get the potential

$$S(V,E) = S^* + C\ln\frac{E - E_H(V) + CT_H(V)}{CT_H(V)} +$$
$$\int_{V^*}^{V} d\eta \frac{1}{T_H(\eta)}\left[\frac{2E_H(\eta) - 2E_* + P_*(\eta - V_*)}{\eta - V_*} + \frac{dE_H(\eta)}{d\eta}\right]. \tag{58}$$

The absolute temperature and pressure are given by the formulas

$$T(V,E) = \frac{1}{K_H(V,E)} = \frac{E - E_H(V) + CT_H(V)}{C}, \tag{59}$$

and

$$P(V,E) = \frac{E - E_H(V) + CT_H(V)}{CT_H(V)}\frac{2E_H(V) - 2E_* + P_*(V - V_*)}{V - V_*} +$$
$$\frac{E - E_H(V)}{CT_H(V)}\frac{d[E_H(V) - CT_H(V)]}{dV}. \tag{60}$$

5. Conclusion

We introduced a novel complete EOS — the $AP\Lambda$-EOS — for hydrocode, which is an operator on three functions of one variable each: $A(V)$, $P(E)$, and $\Lambda(V)$. The model is based on the simple phenomenological assumption that the heat capacity C_V at constant volume is a function of the internal energy density E only. Readers are reminded that the classical ideal case of EOS is based on the assumption that C_V is a function of the absolute temperature T only, whereas the Mie-Gruneisen EOS is phenomenologically equivalent to the assumption that C_V is a function of the entropy density S only (for further information, refer to the publications [5–10]).

We established the explicit formulas permitting recovery of the function $A(V)$, $P(E)$, and $\Lambda(V)$ from different experimental measurements.

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